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COMPARISON OF SEVERAL CODES FOR SOLVING LARGE SPARSE LINEAR SYSTEMS

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ABSTRACT

We report the succesful installation and some comparison of several computer programs for the solution of large, sparse linear systems, of the kind that appear, for instance, in Petroleoum Resevoir Simulation.

COMPARACION DE VARIOS PROGRAMAS PARA LA RESOLUCION DE GRANDES SISTEMAS ESPARCIDOS DE ECUACIONES LINEALES

RESUMEN

Reportamos la instalación exitosa y algunas comparaciones entre varios programas que resuelven sistemas grandes de ecuaciones lineales esparcidas, del tipo que aparece, por ejemplo, en la Simulación de Yacimientos de Petróleo.

1. INTRODUCCION

The numerical solution of (1) $A\underline{x} = \underline{b}$ where A is a nxn, nonsingular, large, sparse matrix is of great importance for many applications. We are particularly interested in Petroleum Reservoir Simulation (PRS), and we present here the preliminary results of a study to find the most appropriate type of method of solution for various problems in that area. In PRS codes, the solution of (1) appears in the innermost loop of a large complex algorithm,[1, 2, 4, 8] and thus it is of fundamental importance to be able to optimize the computation at this level.

Our final objective is to provide a new set of solution options within commercial package currently used by the local Petroleum industry. In this study we have endeavor to install and test under local conditions a number of packages for solving (1).

We have included in this test two direct sparse-matrix solvers and one Iterative Package that contains a number of carefully implemented iterative methods. The results on a simple model problem show that we successfully installed the tested software, and also what are the relative perfomances in two local computing environments.

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Our next step will be to blend these algorithms within an actual simulation package and produce comparisons with actual field data.

We would like to acknowledge the very useful comments of some of the packages authors and thank them for their generosity in making them available to us, namely Professors S. Eisenstat, D. Kincaid and Dr. I. Duff. We would also like to express our appreciation to Dr. Carlos Espinoza for giving us the opportunity of using our Numerical Analysis knowhow in the area of PRS.

2. SOFTWARE EMPLOYED

We have considered in these comparisons both direct and iterative methods. The direct sparse packages used were:
YSMP (Yale Sparse Matrix Package;[6] Uncompressed storage scheme version) and
MA28 (AERE Harwell[5]).

For the iterative methods we used ITPACK,[10] which is in itself a collection of seven different methods.

Given (1), YSPM productes a sparse decomposition A = L D U, and solves successively the three linear systems:

$$L\,\underline{y} = \underline{b},\ D\,\underline{z} = \underline{y},\ U\,\underline{x} = \underline{z},$$

using the standard scheme for sparse solvers:
a) Symbolic factorization
b) Numerical factorization
c) Numerical solution

This version of the package does not produce an ordering atomatically, but rather accepts one input by the user. As it is well known, given a reordering of the rows and columns of A via permutation matrices P, Q, the steps mentioned above will be performed on PAQ instead of A. The objectives of «good» orderings are to reduce fillin (i. e. new non-zero elements produced during the decomposition), to improve stability (pivoting), and/or to reduce the number arithmetic operations needed in the numerical solution.[7,8]

On the other hand, MA28 block-triangularizes A by means of Tarjan's algorithm,[9] and then it reorders each diagonal block using Markowitz's criterium[3] with a threshold for improved stability.

On the iterative side, ITPACK contains seven Fortran subroutines for solving large, sparse, symmetric, positive definite or mildly nonsymmetric linear systems. Basic iterative methods, namely: Jacobi, Successive over-Relaxation (SOR), symmetric SOR, and the reduced system method are combined, where possible, with acceleration procedures such as Chebyshev semi-iteration and conjugate gradients for rapid convergence.

Automatic selection of the acceleration parameters and the use of accurate stopping criteria are the major features of this package.

3. NUMERICAL RESULTS

The model problem used was

$$\triangle u(x, y) = 0 \text{ in } (0,1) \times (0,1)$$
$$u(x, y) = 1 + xy \text{ on the boundary,}$$

discreatized by the standard five-point formula on an uniform mesh

TABLE I

* NOT POSSIBLE DUE TO STORAGE LIMITATIONS.

PACKAGE	*METHOD*	*TIME (SEC.)*		*EXACT*	*DIG.*	*STORAGE*		*ITERATIONS*	
		20 × 20	*40 × 40*	*20 × 20*	*40 × 40*	*20 × 20*	*40 × 40*	*20 × 20*	*40 × 40*
I	JACOBI CG	17	77	5	3	1692	6230	62	73
T	JACOBI SI	22	100	5	2	722	3042	108	110
P	SOR	11	84	5	361	1521	72	110	
A	SSOR CG	10	61	6	3	2234	9166	17	20
C	SSOR SI	11	50	6	3	1805	7605	26	22
K	RS CG	11	35	5	3	1025	3879	31	39
	RS SI	16	69	5	3	541	2281	60	77
MA28	SPARSE ELI	65	*	8	*	34000	*	–	–
YSMP	NATURAL ORDER	21	*	11	*	42000	*	–	–
	ORDER D4	10	200	11	11	14000	100000	–	–

(x_i, y_j) given by: $x_i = ih$, $y_j = jh$, $h = 1/n$, $i, j = 0, \ldots, n$.

Two different orderings were used:

a) *Natural ordering,* in which the mesh is traversed row-wise, from bottom to top;

b) D4,[8] where the mesh is traversed diagonal-wise in checker-board fashion, from the lower left-end corner to the top right-end one.

Ordering b) is commonly used in PRS.

The computer used was a Burroughs 6700.

4. CONCLUSIONS AND FUTURE WORK

From the results in § 3 we can observe the following:

i) Direct methods are more precise than iterative ones. This is one of the important arguments in favor of iterative methods in case where convergence is not a major problem, and where only low accuracy is desired in the numerical solution.

However, there are situations in PRS where one desires to model laboratory experiments, for which high precision measurements are available, and therefore more precise numerical solutions are desired. These problems are also usually smaller than field problems, and thus they may constitute an area where direct methods could be more efficient.

ii) Direct methods require more storage and more CPU time than iterative methods. The difference is larger the larger A becomes, as it will be the case in three dimensional problems and/or systems of PDE'S, like the ones arising in multiphase simulation.

iii) Within the direct methods, YSMP gives the best results using the ordering D4. This points to the fact that knowing a priori a good ordering is more efficient than trying to produce one automatically as in MA28. Obviously, MA28 is much more general and versatile than a program that does not produce its own ordering. Also it would be interesting to include in our comparisons the newer generation of Sparse codes out of Harwell, and the version of the YALE code that produces automatic ordering.

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