Acta Científica Venezolana 38: 676-682, 1987

NUMERICAL METHODS FOR INVERSE PROBLEMS IN PETROLEUM EXPLORATION

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ABSTRACT

Seismic prospecting is one of the important tasks in modern oil exploration. In recent times, with the advent of more powerful computing machinery, three-dimensional imaging of the earth's interior has become more common. There is, consequently, a great need for the development of algorithms and software that can process the huge volumes of data collected in three-dimensional reflection or refraction surveys, and can increase the resolution of the geological and lithological images produced, especially in complex situations. In this paper we present some of the current and recent work done on this area, including a partial survey of the relevant and very extensive bibliography.

METODOS NUMERICOS PARA PROBLEMAS INVERSOS EN EXPLORACION PETROLERA

RESUMEN

La prospección por métodos sismicos es una de las tareas importantes de la exploración petrolera moderna. En tiempos recientes y debido a la aparición de computadores cada vez más poderosos, se ha hecho más común el tratar de producir imágenes tridimensionales del interior de la Tierra.

En consecuencia, hay una gran demanda de nuevos algoritmos e implementaciones que puedan procesar los grandes volúmenes de datos que se recogen en levantamientos superficiales y que ayuden también a mejorar la resolución de las imágenes geológicas y litológicas que se producen, especialmente en el caso de regiones complejas.

En este trabajo presentamos algunas contribuciones recientes en este área, incluyendo una revisión parcial de la extensa bibliografía existente.

INTRODUCTION

We would like to report on some ongoing research and development in the area of the title, surveying some of our results of the past few years.

The long term goal of this research is to produce integrated numerical methods for solving some of the inverse problems found in seismic prospecting and other areas of geophysics. We are especially interested in developing efficient numerical techniques for the imaging of strong reflectors and the recovery of material properties in three-dimensional complex geologic regions.

Imaging the Earth's interior is one of the primary tasks in oil exploration, reservoir management, geothermal prospecting, mapping of coal seams, earthquake seismology and engineering, and other applications. Reflection and refraction surveys are performed on the earth's surface or down boreholes, and records of the returned signals are made along lines or two-dimensional arrays of geophones. In earthquake seismology, fixed or portable stations sense seismic activity, and as a consequence, extensive records of local and teleseismic events are recorded.

Most mathematical modeling of this type of data have been restricted in the past to two-dimensional slices, but in recent years there has been growing interest and need to consider the full three-dimensional problem. In its ultimate complexity this is one of the large problems in scientific computing today.

The forward problem requires the solution of the elastic wave equation in inhomogeneous, three-dimensional media. Simulations are performed by means of finite difference or finite element solvers, and they are very computationally demanding. An alternate technique, more economical, although not as comprehensive, is ray tracing.

The problem of interest here however, is the inverse one **Given recordings of scattered elastic waves, obtain a mathematical model of the structure that produced them.**

Since the pioneering work of Backus and Gilbert[18] there has been extensive research and development on the solution of inverse problems in seismology. A detailed account of the theoretical and practical issues and an extensive reference list can be found in Aki and Richards[19]; see also Bleistein[20] and Claerbout.[21] Chavent[22] and Kravaris and Seinfeld[27] discuss the problem from the point of view of the identification of distributed parameter systems.

Most of the work done so far has been limited by the complexity and computational demands of the problem. Approximations and simplifications are made, either in the dimension or idealization of models considered. The most advanced method in the public domain seems to be that of Aki, Christoffersson and Husebye[24] (see also Aki and Lee[25]), which considers three-dimensional inhomogeneous models, with the velocity assumed to be piecewise constant in parallelepipeds. Even in recently published three-dimensional work[23], only constant velocity layers with plane interfaces are considered. Seismic tomography methods are closely related[31,32], although the works more directly connected to what we are proposing are those of Gjoystdal and Ursin[28]. See also.[29,30]

In [33] we have considered various aspects of these inverse problems in more genral earth models, namely:

(a) Automatic three-dimensional two-point ray tracing in layered regions with curved interfaces.

Algorithms for homogeneous and inhomogeneous media were developed and successfully implemented. The automatic and efficient tracing of families of rays joining a source with a

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This research was partly supported under NSF Grant ISI-8560154

Recibido: 31/3/87
Aceptado: 5/6/87

number of receivers and having the same signature (ordered sequence of regions and interfaces traversed) was attained, by combining shooting and two-point techniques with continuation procedures. Special attention was devoted to bifurcation phenomena, both in the ray tracing as well as on its effect on the sensitivity of the inversion procedures.

(b) Given a set of arrivals, presumed to have reflected from a deep interface, we have developed procedures for imaging the reflecting interface by using ray tracing and a posteriori least squares fitting techniques. The piecewise constant velocity curved layer case was implemented, and a procedure was also described and implemented to estimate the layer material properties from peak amplitude data.

(c) The procedures of (b) are fast, low accuracy techniques, valuable to generate preliminary models. We have also developed dynamic inversion algorithms that involve ray tracing within an optimization loop, to improve upon these preliminary models by using arrival time data. These procedures are more time consuming, and as a consequence we have transported successfully the modules developed in a PRIME 2250 minicomputer to the CRAY XMP/48 at the NSF-UC-San Diego Supercomputer Center. The hundred fold speedup obtained indicates that these procedures can be utilized interactively on current supercomputers, even before careful vectorization or multiprocessing have been included.

2. DATA, MODELS AND RAY TRACING

Explosions, vibrators or air guns, produce signals that travel as elastic waves through the Earth's interior. These signals are reflected, refracted or diffracted by structural and material features, and return to the surface to be recorded on various types of devices. Similarly, seismological stations record automatically seismic activity, both local and at long distances. Earthquakes trigger signals that travel again as elastic waves through the Earth's interior. With differences in scale and emphasis, these two typical mechanisms provide a wealth of data that can be used: (1) to infer underground material properties, (2) to map material discontinuities, (3) to locate the source and time of occurrence of earthquakes or explosions, (4) to model source characteristics. All these are inverse problems, and the aim of the proposed work is to contribute to their efficient numerical solution in three-dimensional, geologically complex regions.

The data sets we want to consider consist in all cases of time histories of velocity or acceleration at a number of receiver locations. From one to three components of the motion may be recorded. In the latter case, we will have available not only waveforms and times of signal onset, but also a record of the direction of arrival of the incoming wave. Preliminary modeling and interpretation experience helps in the identification of coherent signals associated with particular paths through the medium. In what follows, we will speak of arrival times and peak amplitudes of such signals, like those corresponding to primary P and S waves and their various multiples and conversions.

A three-dimensional elastic, isotropic medium will be described either by its Lamé parameters, or more commonly by the velocity of propagation of compressional and shear waves:

$$v_P(\underset{\sim}{\mu}),\ v_S(\underset{\sim}{\mu}),\ \underset{\sim}{\mu} = (x,y,z)^T.$$

Density: $\zeta(\underset{\sim}{\mu})$, attenuation: $Q(\underset{\sim}{\mu})$, porosity: $p(\underset{\sim}{\mu})$, may also be quantities of interest to be recovered from the measured data. We will assume in what follows that these quantities are smooth functions, at least $C^2(\Omega)$, except at a finite number of two-dimensional material interfaces:

$$R = \{\underset{\sim}{\mu} \in \Omega : f(\underset{\sim}{\mu}) = 0\},$$

where they may have jump discontinuities.

Such a medium will be called <u>piece-wise smooth</u>, and the interfaces will be referred to also as <u>strong reflectors</u>. Variations and generalizations of these techniques may also be useful in anisotropic media.

<u>Example</u> 1: Ω = unit cube; R_i, $i = 1,\ldots,r$ non-intersecting plane sections.

$$v_{P_i}(\underset{\sim}{\mu}) = c_i, \quad i = 1,\ldots, r, \quad c_i \text{ constant.}$$

This is a very simple type of model, describing a piece-wise homogeneous medium with plane dipping interfaces.

<u>Example</u> 2: Ω = unit cube partitioned in subregions σ_j whose boundaries consist of reflectors R_{ji}.

Within each sub-region material properties are smooth functions of position.

Obviously, between Examples 1 and 2 there is a whole spectrum of ever increasing model complexity.

In principle, our current code is capable of tracing rays through media of type 2, provided that a topological description of the desired path and a reasonable initial guess are provided. However, considerable effort will be necessary in order to facilitate and automate this task so that it is appropriate for an inversion procedure. A number of interesting issues arise connected with the fact that the two-point ray tracing problem can have multiple or no solutions of a given type. Since we will be solving families of closely associated problems by tracing rays from a given source to a number of receivers, this points naturally to continuation procedures and bifurcation diagrams. Receiver to receiver coherence is an important tool in trying to provide economical solutions. In [33] we have automated most of this task for general curved layered media.

We have also developed earlier a fairly successful 2D code for media of type 2 with piecewise constant velocities. In [12], we have addressed and answered some of the questions stated above. We feel that most of the useful techniques selected there can be generalized to the present case. Among the most important of those techniques we can mention:

(a) Use of shooting as a starting and re-starting procedure.

For given initial directions, rays are traced from the source through the medium until they arrive to the free-surface near a receiver. This path, appropriately modified, is used as an initial trajectory for the 2-point solver. For this purpose we require an initial value code coupled with a root finder to locate the intersections of the ray with the interfaces. It could be argued that shooting is all one needs. However, the pitfalls of shooting are well known,[26] and we have indeed observed in many examples[8,11,12,16] the high sensitivity to variations in initial angles that these problems have. The coupled algorithms provide an ideal blend of the best features of each individual technique.

(b) Receiver and initial direction continuation

Due to strong possibility of multiple arrivals af a given type, receiver continuation, as it has been advocated recently,[1,2,6] will not be sufficient to provide all the arrivals without additional techniques. Usually, the iterative methos employed to solve the nonlinear ray problem will fail near bifurcation or turning points. One could plausibly implement a complete bifurcation diagram generator for this problem, but we have found a much simpler solution that is quite effective and works well in the present situation. Basically the technique consistes of keeping track of the initial directions that have been inspected, either by shooting or 2-pointing, and using shooting to re-start whenever the 2-point scheme fails. Essentially this amounts to using receiver continuation within coherent branches of solutions, and using shooting to get started in new branches, A control screen with a hierarchical family of nested windows of varying resolution helps in optimizing the number of rays that need to be traced. We believe that similar techniques could be valuable in other applications, and we emphasize that here they are specially natural and intuitive.

In describing the model we may use data structures that contain connectivity information. In this form it will be possible for the shooting module to provide automatically the ray signatures (i.e., an ordered sequence of identifiers for the media and reflectors traversed) which are need in the 2-point module, thus relieving the user of that responsibility, even in the most general geometries.

3. SEQUENTIAL RECOVERY OF REFLECTORS FROM TIME OF ARRIVAL DATA: A TWO POINT APPROACH

Given a collection of time histories it is often possible to pick up coherent arrivals that indicate the presence of a strong reflector. In this section we will show how we plan to use 2-point ray tracing to map such a reflector to its actual position within the geological region Ω, using the time data. This task is usually referred to as time-to-depth migration.[5,7,9,12]

Let, ideally, $T(x,y)$ be the arrival time at $(x,y,0)$ of a signal that follows a trajectory through a known medium, touching once the unknown reflector R at a point $P(x,y)$. For simplicity, assume that there are no other discontinuities. It turns out that

$$\gamma = (\partial T/\partial x,\ \partial T/\partial y,\ -1)^T$$

is the direction of arrival of the signal at $(x,y,0)$. Thus we can pose the following two-point problem:

Solve the ray equations:

$$\frac{d}{ds}\left[u(\underset{\sim}{\mu})\frac{d}{ds}\underset{\sim}{\mu} \right] - \nabla u = 0 \tag{1}$$

where $u(\underset{\sim}{\mu}) = 1/v = 1/v(\underset{\sim}{\mu})$ and s is arc length along the ray;

Subject to the boundary conditions:

$$\underset{\sim}{\mu}(0) = \text{source}; \qquad \underset{\sim}{\mu}(S) = \text{receiver} \quad (S \text{ is the unkown total arc length})$$
$$\underset{\sim}{\mu}'(S) = \underset{\sim}{\delta} \tag{2}$$

Continuity at the unknown point $P = \underset{\sim}{\mu}(S_p)$, which corresponds to an unknown value S_p of s. (3)

We need in all 14 boundary conditions to account for the six differential equations, the discontinuity, and the two unknown parameters S, S_p. We have already provided 12.

The two remaining ones are:

$$\| \underset{\sim}{\mu}'(0) \|^2 = 1, \text{ arc length condition}^{14} \tag{4}$$

and

$$\int_0^S u(\underset{\sim}{\mu})ds = T(x,y), \text{ travel time condition} \tag{5}$$

Thus the problem is well posed and can be solved by the code PASVA4 [8,11], once we put it into first order form and cast (5) as an additional differential equation:

$$T' = u(\underset{\sim}{\mu}), \quad T(0) = 0, \quad T(S) = T(x,y), \tag{5'}$$

plus continuity at the interface.

The solution to this problem will give us the point P on the unknown reflector: $P = \underset{\sim}{\eta}(Sp)$, and also the tangent plane of the reflector at P. This follows from the fact that the bisector of the incoming and reflected ray directions is the normal to this tangent plane. Thus each observation provides three pieces of information about the unknown reflector. Using a number of well placed source-receiver pairs we should be able to illuminate the whole reflector with enough redundancy, so that with an appropriate parametrization and an a posteriori least squares computation a good image is obtained. Again, there are important practical issues that need to be addressed in order to convert this idea into a usable tool, and we have started to explore some of them in our early 2D work, and in [33] for 3D piecewise homogeneous curved layered media.

A set of data corresponding to coincident source-receiver pairs is called a normal incidence time section. This usually simulates what is called in geophysical data processing a stacked section, where the original data has been summed up in order to improve the signal-to-noise ratio. This type of processing has a number of pitfalls in complex geological regions, but we will not be concerned with them here.

For a normal incidence time section we can use an initial value approach for migration. In fact, given the source-receiver position and the initial direction of travel provided by the time surface, we need only to shoot rays through the structure for the allotted time T(x,y). The tip P of each ray should lay on the unknown reflector, while the ray direction is the normal to the tangent plane at P. This can in turn be posed as a two-point boundary value problem, which may be more stable than the initial value formulation.

A blend of these migration algorithms with the least squares approach to be discussed in the next section should permit the step-by-step, interactive creation of a model that integrates and attempts to explain all the available data. In addition this approach would provide information on confidence regions and sensitivity of the model parameters.

In[33] we have also shown how to recover material properties (i.e., P and S velocities v_P, v_S, and density ζ), in the case of homogeneous layers, by using a nonlinear least squares fit coupled with the calculation of elastic reflection transmission coefficients via Zoeppritz formulae. This was achieved by first correcting peak amplitudes by geometrical spreading and previous interfaces reflection-transmission coefficients.

For a given parametrization of the medium material properties we think that this idea can be extended to nonhomogeneous materials, at least for moderate interlayer variations in properties. Recent results of Bleistein seem to indicate that it is possible to obtain reasonable estimates of these quantities by using multiple shots, contrary to earlier indications that only impedances could be appropriately resolved.

4. NONLINEAR LEAST SQUARES DYNAMIC INVERSION

The physical quantities mentioned in Section 2, that describe a geological medium Ω, were assumed to be piecewise smooth functions defined in Ω. In order to obtain estimates for them by a numerical inversion procedure, we need to discretize, i.e., to consider a finite dimensional representation. The choice of parametrization will be crucial in terms of our ability to represent "the real Earth". Issues about resolution and information content of the given data set will have to be considered.[18]

Methods in use differ in the type of discretization employed and also on the time at which the discretization is effected.[22] We have chosen to consider "early" discretization. That is, for any function to be retrieved, like v_P, v_S, ζ, Q, etc. we will associate a finite dimensional parametrization. We are quite open minded at this time with respect to which form of parametrization is to be preferred although we have some pointers based on our previous experience. In any case, there are some prerequisites that any parametrization must fulfill in order to be useful for our purpose:

(i) It should at least produce functions in PC^2 (piecewise C^2).

(ii) It should be as concise as possible (i.e., low dimensinal).

(iii) It should be reasonably easy to produce and evaluate.

This problem is actually akin to surface representation from scattered data. Earth models are seldom built from scratch, and we can think of our methods as improving upon a preliminary model. A key to success will be the ability to integrate all available information, which may come in a number of different forms, such as: well-logs, extrapolated geological observations, contour line maps, topographical maps, etc.

In two dimensions we have been using both interpolatory and least squares cubic splines for the representation of general curved interfaces and one-dimensional velocity models; also analytic representations have been used when warranted. For the three dimensional case, least squares approximations with tensor products of cubic splines basis seems an adequate first choice, although other candidates should be considered, like parametric splines.[34]

These tasks sound somewhat mundane, but they are really at the heart of the whole scheme. Ingnoring them may not impair the description and theoretical aspects of the algorithms for the main task, but it will certainly detract from the possibility of testing these algorithms in any real applications.

From the ray tracing equations, we see that the velocity of propagation affects the ray trajectories and the travel time through the differential equations, while the material discontinuites show up only in the boundary conditions. Although we have not discussed it, densities and velocities will also affect our calculation of amplitudes by geometrical spreading and reflection-transmission coefficients, which will become relevant if we consider peak amplitudes as further pieces of data.

Thus, having assumed a parametrization for, say, $v_{P_i}(x,y,z,\underset{\sim}{\eta}_i)$ $i=1,\dots,Nv$, and interfaces $f_j(x,y,z,\underset{\sim}{\lambda}_j)=0$, $j=1,\dots,NI$, we have a nonlinear least squares problem for travel time data:

$$\min_{\underset{\sim}{\eta}_i,\ \underset{\sim}{\lambda}_j} \sum_{k=1}^{Nobs} \left[T^k_{obs} - T^k_{comp} \right]^2$$

where k runs over all the observations. From the expression for T^{comp}:

$$T^{comp}(\underset{\sim}{\eta}, \underset{\sim}{\lambda}) = \int_0^S u(\underset{\sim}{\mu}_k(s; \underset{\sim}{\eta}, \underset{\sim}{\lambda}); \underset{\sim}{\eta})ds$$

we see that the dependence of the functional to be minimized on the model parameters is quite complicated, since it involves tracing rays between all the NObs source-receiver pairs (in order to obtain $\underset{\sim}{\mu}_k$).

Starting from an initial model $[\underset{\sim}{\eta}_i^{(0)}, \underset{\sim}{\lambda}_j^{(0)}]$ we will compute corrections by some method based on linearization. Depending upon the size and characteristics of the particular problem, we will use either a direct or an iterative method for solving the resulting linear least squares problems. It does not seem that at this time exists adequate software in the public domain that will effectively consider all the special features of this problem. Depending upon the complexity of the medium and data sets, a variety of least squares problems will arise. For media that can be described by a relatively small number of parameters and for which the available data is also of moderate size, we can use incore direct methods. As a matter of fact, SVD based algorithms would be quite adequate, since they will bring out at each iteration as much information as is present in the expensive Jacobians of the residual vectors. For larger problems, iterative methods will become necessary and conjugate type iterations are likely candidates for consideration.

Each observation provides one row of the matrix of the linearized problem, and only those parameters corresponding to sub-regions "seen" by the ray will be nonzero. Thus we can expect some kind of loose block structure to emerge, reflecting the fact that families of close by rays will travel through the same subregions; otherwise the coefficient matrices will have a fairly general sparse structure.

The solution of nonlinear least-squares problems subject to nonlinear constraints is the subject of much active research. Based on recent literature, our view is that an "off-the-shelf" method is unlikely to be the most effective choice for the problems in this application, but that there is great promise for the development of specially tailored algorithms.,

Even for purely unconstrained nonlinear least-squares, certain kinds of problems remain extremely difficult to solve (see, e.g., Al-Baali and Fletcher[35]; Fletcher and Xu[37]; Salane[42]).

Most of these problems are nearly rank-deficient (as are those arising in this work), and every general strategy devised thus far becomes inefficient, or may fail, in some circumstances. However, because the problems in this project all arise from the same basic application, the hope is that effective methods can be developed by systematic exploitation of the structure of the ill-conditioning (as in solving the ill-conditioned systems of equations that arise in penalty function methods; see, for example, Gill, Murray and Wright[43]). An extremely promising strategy is to add linear constraints designed to confine the ill-conditioning to a certain subspace.

For large-scale nonlinear least-squares, recent developments in nonlinear approaches to linear programming (Karmarker[40]) that have led to great interest in large-scale linear least-squares, may be relevant. Much progress has been made in the use of "approximate" Cholesky factors (e.g., Gay[38]), in the construction of effective preconditioners for the conjugate-gradient method (see, e.g., Gill at al.[39]), and in the application of iterative methods to specially ordered submatrices (Dennis and Steihaug[36]).

When faced with a constrained nonlinear least-squares problem, one key issue is the relative importance to be associated with the least-squares structure of the objective function on the one hand, and with the treatment of constraints on the other. With a dense purely linear problem (linear least-squares with linear constraints), a standard active-set method can be implemented effectively. In the presence of nonlinearities, however, complex tradeoffs need to be explored, such as the effect of retaining a separate least-squares term in the approximate Hessian of the Langrangian within a sequential quadratic programming method. (For a discussion of this issue in an exact penalty method, see Mahdavi-Amiri and Bartels[41]).

Finally, there are some aspects of the problem that may be amenable to generalized separable nonlinear least squares techniques[3,4,17], as we have already shown in [33].

CALCULATION OF JACOBIANS

To obtain the coefficient matrix of the linearized least squares problem, we will need to compute $\partial T/\partial\sigma$, for each $\sigma \in \{\underset{\sim}{\eta},\underset{\sim}{\lambda}\}$ and for each observation.

From (5') we see that this will require both $\partial u/\partial\sigma$ and $\partial\underset{\sim}{\mu}_k/\partial\sigma$. The first derivative can be obtained from the expression for $u(x,y,z;\underset{\sim}{\eta})$. By differentiating the ray equations (1) with respect to σ along the given ray $\underset{\sim}{\mu}(s)$, we see that $\underset{\sim}{\zeta} = \partial\underset{\sim}{\mu}/\partial\sigma$ satisfies a linear system of ODE's.

In order to simplify the presentation of this system, we write (1) in first order form, by introducing the new variables $\underset{\sim}{w}(s) = u\, d\,\underset{\sim}{\mu}/ds$:

$$d\underset{\sim}{\mu}/ds = v\underset{\sim}{w} \tag{7}$$

$$d\underset{\sim}{w}/ds = \nabla u \tag{8}$$

$$dT/ds = u. \tag{9}$$

Differentiating we get

$$d\zeta/ds = [\ \langle \nabla v, \zeta\rangle + \partial v/\partial\sigma]\ d\underset{\sim}{w} + v\underset{\sim}{\omega} \tag{10}$$

$$d\underset{\sim}{\omega}/ds = (Hu)\underset{\sim}{\zeta} + \nabla(\partial u/\partial\sigma) \tag{11}$$

$$d\tau/ds = \langle \nabla u, \zeta\rangle + \partial u/\partial\sigma \tag{12}$$

and corresponding boundary conditions.

In (10-12) we have put $\omega = \partial w/\partial\sigma$, $\tau = \partial T/\partial\sigma$, $Hu = (\partial^2 u/\partial\mu_i\partial\mu_j)$, i, j = 1, 2, 3, for the Hessian matrix of $u(\underset{\sim}{\mu})$, and $\langle\, ,\, \rangle$ indicates vector inner product.

These turn out to be the linearized ray equations used in discretized form by the code PASVA4 to solve the nonlinear problem iteratively. The only difference is that they have the special forcing term $\underset{\sim}{r}$:

$$\underset{\sim}{r} = \begin{bmatrix} v\omega + \partial v/\partial\sigma w \\ \nabla(\partial u/\partial\sigma) \\ \partial u/\partial\sigma \end{bmatrix} \tag{13}$$

Upon exit from the ray code with a converged ray $\underset{\sim}{\mu}$, we have the discrete Jacobian of the differential equations in LU form. Thus the discrete solution of (10-12) requires only to compute (13) on the mesh, and to perform a back substitution - a very unexpensive process. In this form, PASVA4 and the two-point approach facilitate the gradient computation for the functional in the least squares problem. We should mention here that the same technique is already used in the calculation of geometrical spreading amplitudes and in the implementation of Euler continuation for the bifurcation studies.

6. SUPERCOMPUTERS AND ADVANCED ARQUITECTURES

Clearly, realistic three dimensional interactive modeling is a highly intensive computational task. So far, this has limited its widespread application and the amount of resolution and model complexity that can be handled. However, with the tremendous advance in super-computers and parallel arquitectures the situation is rapidly changing, and if we add a modicum of intelligence to our algorithms considerable improvements should be feasible.

In our preliminary work in[33] we have used a CRAY XMP/48 super-computer to great advantage, and we have also foreseen the use of parallel arquitectures. With that purpose in mind, we have strived to modularize the algorithms, emphasizing any inherent parallelism, specially at the subroutine level. This structure should be highly transportable to a number of message passing systems. In each one of the large tasks classes in which we have subdivided the problem there are good opportunities for simultaneity, as we indicate in what follows.

Ray tracing can be naturally accomplished in parallel without loosing the sophistication of our current automatic algorithms. On one side, for each interface and class of rays one can fragment the region with our multi-window procedure and assign different windows (corresponding to different sets of arrivals or folds of the time surface) to different processors. On the other side, we can simultaneously initiate a ray trace in more than one layer, or for more than one class (i.e. rays with different signatures, like multiples, diffractions, refractions, etc.). In general, any independent family of rays can be assigned to a separate processor, preserving all the advantages of the current automatic ray tracing procedure. Of course, this will necessitate reasonably powerful nodes, so it may not be appropriate for some highly parallel machines with weak nodes. On the other hand, inter-node communication is not necessary since all what is needed is to initiate the task in each processor and collect the computed rays at the end.

As a matter of fact, this very same scheme is applicable to the ray-tracing involved in the time-to-depth migration procedures, where even the least squares fitting can be effected in parallel for each independent patch. Since in this procedure we must work on one interface at a time, this technique may be applied to deal with more extense or complicated regions, when a sufficiently large number of independent sub-patches are present. Observe that we are thinking of systems that facilitate the dynamic spawning of tasks and processor allocation, as most adequate for our purposes.

In the case of the combined static-dynamic layer peeling algorithm, a high degree of parallelism can be achieved by assigning different layers, and/or sub-patches to different processors. The administration of all this complexity can be kept reasonably under control by including just a few parameters, which will regulate precedence and identify uniquely the various families of rays, interfaces and patches. Load balancing may be trickier, although if we have more tasks than processors we can plausibly achieve a fairly good load balance, at least in the middle game.

There are other parts of the algorithms that can benefit from fine grain parallelism, and where high efficiency can be achieved in other arquitectures that are more geared to this type of tasks. These include some basic and standard linear algebra tools, like linear equation solving, matrix operations, least squares calculations, and also seismogram generation and other signal processing tasks. The special linear equation solver used in PASVA4, which is one of the time consuming parts of the ray-tracing, can be made more efficient by careful re-tooling to increase vectorization.

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