Optimal Design of steel frame structures

J. Isenberg, V. Pereyra, and D. Lawver

Weidlinger Associates, 4410 El Camino Real, #110, Los Altos, CA, U.S.A.

Abstract

We consider the optimal design of steel structures. Given a preliminary design, we attempt to minimize the total weight of the structure subject to various loads and constraints. We combine finite element structural analysis codes with nonlinear programming optimization ones. Several examples are given.

Key words: Optimal structural design; steel structures.

1 Introduction

In many industrial problems design optimization is achieved through a trial and error process that uses the expertise of an experienced engineer. In such a process, different parameters are manipulated and either numerical or graphical output data is examined to choose between designs.

When the number of design parameters increases this becomes a tedious and sometimes impossible task. Thus an automation of this expert based process is desirable, both because it may help do without scarcely available experts and also because it may allow mechanization of the process.

In order to setup such a mechanized process we need to formulate the problem in an appropriate way:

- The process needs to be modelled.
- The models need to be parametrized.
- A simulation tool has to be available.
- A goal and a set of constraints have to be defined.

To optimize the design of steel structures we choose as our initial optimization goal the minimization of the weight of the structure, subject to load and other constraints. We consider gravity and wind loads to start. Then we also include stress constraints according to code requirements and some additional practical (i.e., expert driven) constraints.

Given an initial design that establishes the topology of the structure, we consider as parameters for the optimization the cross-sectional area (A) and bending moments (M) of groups of members. This limits the number of different member types, and thus makes the design more practical.

Let N be the number of different groups of members, N_c the number of constraints and N_s the number of currently active (i.e., violated) constraints. We define the weight factor for the group of members i as

$$w_i = \sum_j \rho L_{ij},$$

where L_{ij} is the length of member j in group i, and ρ is the density of steel. The total weight is our goal functional:

$$\Phi(\mathbf{A}) = \sum_{i=1}^{N} w_i A_i,$$

where A_i is the cross-sectional area of group member i. Thus the problem is:

$$\min_{\mathbf{A}} \Phi(\mathbf{A}),$$

$$\textit{subject to} \;\; g_s(\mathbf{A}, \mathbf{M}) \leq g_s^u, \quad s = 1, ..., N_c.$$

For example, the constraints g_s can either represent limit drifts of the different stories when subject to wind or earthquake loads, or they can represent limit values of the parameters, or they can represent limits on allowable stresses. For simulating the elastic response of the structure we call the finite element code FLEX [8]. Its output is used to evaluate the goal functional and the constraint residuals.

2 The optimization problem

Let $L(\mathbf{A}, \mathbf{M}, \lambda)$ be the Lagrangian of the problem, defined as:

$$L(\mathbf{A}, \mathbf{M}, \lambda) = \sum_{i=1}^{N} w_i A_i + \sum_{s=1}^{N_s} \lambda_s (g_s(\mathbf{A}, \mathbf{M}) - g_s^u),$$

where N_s are the active, or violated constraints.

The Karush-Kuhn-Tucker (KKT) optimality conditions (see [4]) are obtained by setting the gradient of the Lagrangian with respect to the parameters to zero:

$$\frac{\partial L}{\partial A_i} = w_i + \sum_{s=1}^{N_s} \lambda_s \frac{\partial g_s}{\partial A_i} = 0, \quad i = 1, ..., N,$$

$$\frac{\partial L}{\partial M_i} = \sum_{s=1}^{N_s} \lambda_s \frac{\partial g_s}{\partial M_i} = 0, \quad i = 1, ..., N.$$

Since the goal functional is linear in the parameters, the constrained minimum has to occur at a constraint boundary (or intersection of constraint boundaries). Thus, in order to find an optimal point we need to solve simultaneously the KKT conditions and the nonlinear equations corresponding to active constraints, i.e., those satisfied with equality at the solution:

$$g_s(\mathbf{A}, \mathbf{M}) - g_s^u = 0, \quad s = i, ..., N_s.$$

As in any nonlinear programming algorithm, one has to make an educated guess of which constraints will be active at the solution and be able to switch constraints within the iteration as the situation warrants it.

3 Numerical algorithm

In order to solve the optimization problem we propose the following iterative procedure, which is a generalization and liberal interpretation of the one found in [3]:

- If there are N_s violated constraints, select them as candidates for active constraints at the solution; if not, chose the one that is closest to violation. This heuristics is justified by the fact that the goal functional is linear and decreases from the inside to the outside of the feasible region. If there is a clear sign that more than one constraint will be active at the solution, then all the potential active constraints should also be incorporated into the Lagrangian when coming from the interior of the feasible region.
- Rewritting the KKT conditions, we have:

$$f_i = \sum_{s=1}^{N_s} \frac{\lambda_s}{w_i} \frac{\partial g_s}{\partial A_i} + 1, \tag{1}$$

$$f_{N+i} = \sum_{s=1}^{N_s} \lambda_s \frac{\partial g_s}{\partial M_i}. \tag{2}$$

We write a Picard type iteration in order to find a zero of this system of equations. Starting from an initial guess of the parameters, i.e., those corresponding to an initial design, we correct according to the following iterative formulae:

$$A_i^{\nu+1} = A_i^{\nu} + \eta^{-1} f_i, \qquad (3)$$
$$M_i^{\nu+1} = M_i^{\nu} + \eta^{-1} f_{N+i},$$

where η is a parameter chosen to obtain a contracting iteration function (see [5]). This correction contains the unknown Lagrange multipliers. In order to estimate them we resort to solving the active constraint equations.

- To solve the active constraint equations we expand in Taylor series:

$$g_s - g_s^u = J\delta,$$

where J is the Jacobian matrix of the active constraints. Replacing δ by the values from (3), we get:

$$JDJ^T\lambda = -J(1|0)^T - \eta(g_s^u - g_s),$$

where D is a diagonal matrix containing the weight factors w in the first N positions and ones everywhere else. Solving this system of equations for the multipliers and replacing in (3) we can update the parameters.

4 Implementation

The algorithm above was described under the assumption that the parameters could be varied continuously. This is not practical, since AISC W-sections are only available in a finite, discrete, set of shapes. Thus, at every step we must cast our continuous variables into the closest available W-section.

If the constraints are maximum inter-story drift and total drift, given a design, we must perform an analysis of the structure in order to calculate the drifts. We use the Finite Element code FLEX to do that, although any structural code could be used to feed back inputs to the optimization code. In fact, in a following stage we have used the commercial code SAP-2000 on a Windows platform for analysis.

An additional difficulty arises because of the fact that there are no readily available partial derivatives of the drifts with respect to the parameters, which are necessary to formulate and solve the problem. Ideally, we would increment each parameter at a time to approximate the corresponding partial derivative by a finite difference. Unfortunately this is not practical again, because of the fact that W-sections only come in a limited variety, and the closest to a given one usually will have different values for both the cross-sectional area and bending moment.

In order to solve this problem we select for each member two nearby W-sections, providing independent increments for the cross-sectional area and bending moments. We perform analyses of the structure for these modified members, obtaining incremental values of the drifts and other constraints. By solving a 2×2 linear system we obtain approximations to the desired partial derivatives, at the cost of $2N$ analyses of the structure. During this calculation we monitor the residual drifts and if they have decreased, with respect to the minimum obtained previously, we interrupt the process and take that set of W-sections as our next iterate. This has proven to be a very effective sampling procedure that accelerates the overall process.

Another problem, brought up by the casting of the continuous values into the finite set of W-sections, arises when the updated values of the parameters are not sufficiently different from the previous ones, and the closest W-sections turn out to be the same as before. In this case we increment the value of the correction until there is a change, to avoid cycling. Clearly this introduces an ultimate limitation on the precision that can be achieved, since upon convergence the size of the correction will tend to zero.

Finally, following current practice, we do not allow W-sections larger than W14 for the columns of our example frame, an additional type of constraint. In general, additional constraints for ranges of beam and column sizes could be input by the user.

5 An example with only wind and gravity loads

In order to prove the feasibility of the approach, we consider as a simple example a two-story frame (see Figure 1).

We group the columns of each floor so that they will use the same W-section, and thus we have a total of eight parameters for the four different sections. The constraints are:

$$first\ story\ drift \ \leq 1.28,$$

$$second\ inter-story\ drift \ \leq 0.88,$$

$$total\ frame\ drift \ \leq 1.44.$$

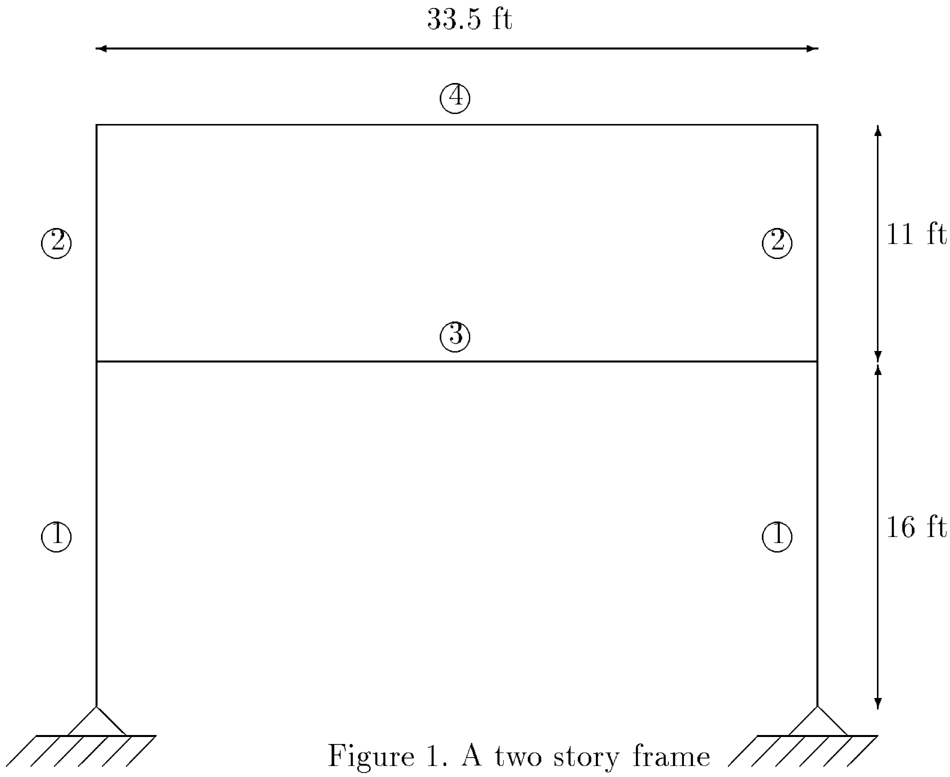

Figure 1. A two story frame

The initial design, based on gravity load preliminary values, uses W14x68 sections for the columns, W24x62 sections for the first cross-beam and W16x36 for the second one. In Figures 2 and 3 we see the evolution of the two drifts and the weight of the structure as the iteration proceeds. The second inter-story drift is of lesser importance than the others and it is satisfied throughout the process.

Thus we see that the initial design does not satisfy all the constraints and needs to be reinforced. This is done until all the constraints are satisfied, at which point we are within the feasible region. We show in Table 2 the evolution of the design throughout the iteration. The meaning of the columns in Table 2 is given in Table 1. The total number of finite element analyses was 18.

6 Stress constraints

The strength of members subject to combined stresses is determined according to the following provisions (see [7]):

$$\frac{f_a}{F_a} + \frac{C_{mx} f_{bx}}{(1 - \frac{f_a}{F'_{ex}}) F_{bx}} \leq 1.0, \qquad (4)$$

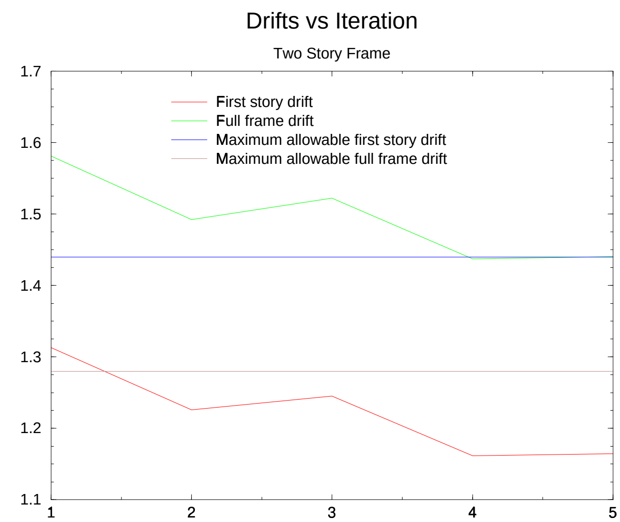

Fig. 2. Drifts for two story frame

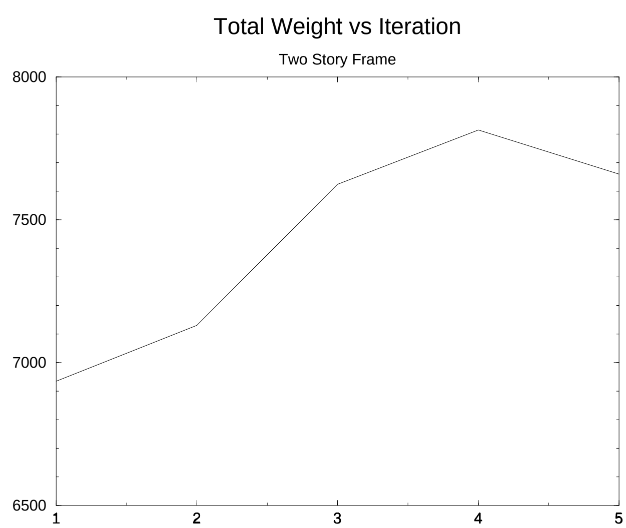

Fig. 3. Weights for two story frame

Table 1
Explanation of headers in Table 2

iter:	Iteration number; 0 is the initial guess
type:	S = reduction in residual drifts during evaluation of Jacobian;
	KKT = full correction;
calls:	Number of FLEX calls in this iteration
weight:	Current weight of the structure
C_1:	W-section for first floor columns
C_2:	W-section for second floor columns
B_1:	W-section for first floor beam
B_2:	W-section for second floor beam
rd_i:	Residual drifts; a negative value means that the drift constraint is violated.
	All positive values mean that the point is feasible.

Table 2
Single bay two-story frame results

iter	type	calls	weight	C_1	C_2	B_1	B_2	rd_1	rd_2	rd_3
0	-	1	6935	w14x68	w14x68	w24x62	w16x36	-0.033	0.61	-0.14
1	S	2	7130	w14x74	w14x68	w24x62	w16x36	0.054	0.61	-0.052
2	KKT	9	7624	w14x74	w14x68	w21x68	w14x43	0.035	0.60	-0.082
3	S	2	7814	w14x82	w14x68	w21x68	w14x43	0.118	0.60	0.0027
4	S	4	7658	w14x82	w14x61	w21x68	w14x43	0.116	0.60	-0.0004

$$\frac{f_a}{0.6F_y} + \frac{f_{bx}}{F_{bx}} \leq 1.0, \tag{5}$$

where we have simplified the expressions for the case of a two-dimensional frame (no y terms) and where:

f_a	Actual maximum axial stress of member (as calculated by FLEX) $= frc/A$
frc	Maximum force in member
A	Cross-sectional area
F_a	Allowable maximum axial stress, simplified to $= 0.4F_y = 14.4$
f_{bx}	Actual maximum bending stress $= b_m/s_x$
b_m	Maximum bending moment
s_x	Elastic section modulus
F_{bx}	Allowable maximum bending stress $= 0.66F_y = 23.76$ (used for compact sections)
F_y	36 ksi for steel
F'_{ex}	$\frac{12\pi^2 E}{23(Kl_b/r_x)^2} = 37332(r_x/l_b)^2$
l_b	Unbraced length in the plane of bending
E	Elastic modulus of steel $= 29000$ ksi
K	Effective length factor $= 2.0$
r_x	Radius of gyration $= \sqrt{I_x/A}$
I_x	Moment of inertia from W-member properties

Putting these quantities into (4,5) we get, for each member, in addition to the three constraints associated with the wind loads:

$$
\begin{aligned}
g_s(\mathbf{A},\mathbf{M}) &= \frac{0.0694 frc_s}{A_s} + \frac{0.03577 b_m}{s_x(1 - f_a/F'_{ex})}, \quad s = 2j,\ j = 2,7\ , \qquad (6) \\
g_s(\mathbf{A},\mathbf{M}) &= \frac{0.0463 frc_s}{A_s} + \frac{0.0421 b_m}{s_x}, \quad s = 2j+1,\ j = 2,7\ ,
\end{aligned}
$$

for a total of $N_c = 15$ constraints. Observe that due to the asymmetric effect of the loads we cannot group the members here, since opposite columns will have different stresses.

7 An example with wind, gravity and stress constraints

We consider the previous two story frame example, but now we impose also stress constraints in all the members. The method, loads and starting design are all identical to the previous example.

We present the results in detail in Table 3 and also in graphic form in Figures

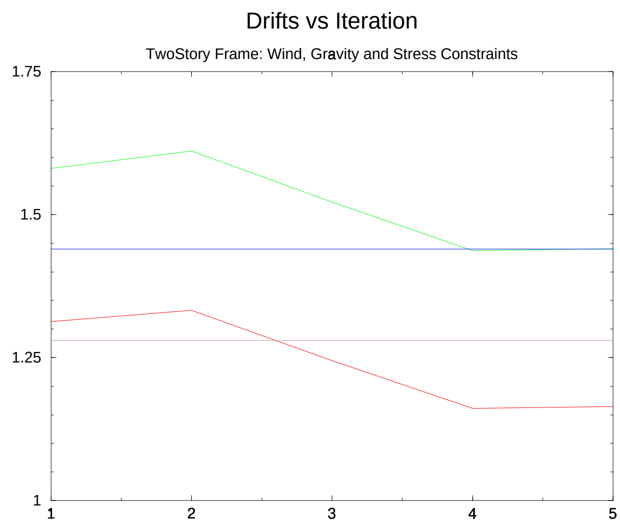

Fig. 4. Drifts for two story frame with wind, gravity loads and stress constraints

4 and 5. This table is slightly different from the previous one, since there are now too many constraints to list all the residuals. Thus, we list only the evolution of the residuals asociated with constraints that become active at some point in the process namely: the first story and total frame drifts.

Table 3
Results for two story frame with stress constraints

iter	type	calls	weight	C_1	C_2	B_1	B_2	rd_1	rd_3
0	-	1	6934	w14x68	w14x68	w24x62	w16x36	-0.033	-0.14
1	KKT	9	7543	w14x68	w14x68	w21x68	w14x43	-0.053	-0.17
2	S	2	7568	w14x74	w14x68	w21x68	w14x43	0.034	-0.082
3	S	2	7814	w14x82	w14x68	w21x68	w14x43	0.119	0.0027
4	S	4	7659	w14x82	w14x61	w21x68	w14x43	0.116	-0.0004

Although the path was somewhat different than for the earlier example, the final design and the total number of analyses are identical. In the previous example there was a sharp decrease in the violation of constraint #3 in iteration 1, while now we have a sharp increase in the violation of constraint #14, and thus the difference in behavior.

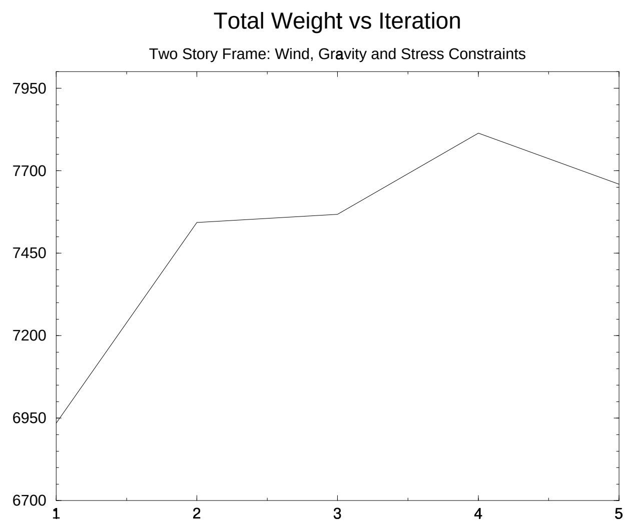

Fig. 5. Weights for two story frame with wind, gravity loads and stress constraints

8 A one-bay eight-story frame

Now we consider a somewhat larger problem and generalize some aspects of our previous code. The problem has been considered in [2], where two optimal solutions are offered: one corresponds to the method they are proposing, based on a genetic algorithm, while the other comes from [6], which used an optimality criterion method.

In Figure 6 we show the structure. The numbers associated with the members indicate the group to which they belong. Members within a group will be kept identical through the optimization in order to avoid unwieldy changes in the structure and limit the number of free parameters.

The only constraint used in the reference above and in these comparisons is the total drift of the structure, which is limited to 2 in. We also limit the maximum size of columns to W20.

We consider the optimal solutions in [2] as starting points for our optimization. Initial 1 corresponds to the solution of Khot et al, while Initial 2 is that of Camp et al. In Table 4 we show the initial and final designs for these two experiments and also the weight of the structure in pounds.

Table 4
Eight story frame. Starting from Khot and Camp designs

Member	Initial 1	Final 1	Initial 2	Final 2
1	W13x34	W16x31	W18x46	W16x40
2	W10x39	W10x30	W16x31	W14x26
3	W10x33	W10x30	W16x26	W14x30
4	W8x18	W10x15	W12x16	W10x15
5	W21x68	W21x68	W18x35	W18x35
6	W24x55	W24x55	W18x35	W18x35
7	W21x50	W21x50	W18x35	W18x35
8	W12x40	W12x40	W16x26	W16x26
Weight	9,220	8,496	7,380	7,060

We also switch now from our finite element code FLEX to SAP2000, a commercial structural analysis code. Additionally, we had to port the optimization code from UNIX to a Windows environment, as required by SAP. The way in which the optimization code interacts with the finite element analysis codes is through system calls that initiate batch execution. Further communication is through files (message passing). Thus, the optimization code updates an initial SAP input file to introduce the changing member sizes, and then reads from an output file the information produced during the analysis of the updated structure.

9 Eight story frame with code constraints

More realistically, we introduce now stress constraints and also two different load combinations. Since each member generates two constraints, there is now a total of 49 constraints.

SAP checks and reports stresses and bending moments at a number of points in each member. Our calculation of the constraint value (6) is done for each pair of values, for every member and every load combination. Then, we pick the worst case as the actual value of the constraint.

For this example we started from a different, stronger design that satisfies SAP stress constraint check (ours is a simplified version). In Table 5 we show the initial and final members and the corresponding weights for the structure. We see that the total weight has been cut by more than 6%.

Unfortunately, when using SAP to check the code constraints, this lighter design violated code. That led us to revise the approximations made in our statement of the code constraints. A more accurate formulation is used in the next section.

10 An exact penalty method

The performance of the method of the previous sections was not what we expected. The method is not robust enough and we do not expect it to scale to larger problems, our main goal in this study. Thus, in this section we explore the use of an exact penalty method, to transform the nonlinearly constrained problem into an unconstrained one.

The idea is to add to the weight functional a penalty term that is a function of the violated constraints. The larger the violation the larger this term will be. There are many flavors of penalty and barrier methods. Following [4], we consider the exact penalty method:

$$min_\alpha \ weight(\alpha) + \rho \sum_{i=1}^{N} |res_i(\alpha)|,$$

where $\alpha = (\mathbf{A}, \mathbf{M})$, and:

$$res_i(\alpha) = g_i^u - g_i(A_i, M_i) \leq 0,$$

(the active constraints.) If α^* is a Kuhn-Tucker point of the constrained problem, then for ρ sufficiently large, α^* is also a minimum point of this unconstrained problem.

We use PRAXIS [1] to solve the unconstrained minimization problem. An important advantage of PRAXIS is that it does not require derivatives. As we indicated above, the code constraint formulation we use needs to be improved.

Table 5
Eight story frame with code constraints imposed

Member	Initial	Final
1	W14x61	W12x65
2	W12x58	W14x53
3	W12x58	W14x53
4	W12x40	W12x40
5	W16x26	W6x9
6	W16x26	W16x26
7	W16x26	W16x26
8	W16x26	W16x26
Weight	10,728	10,077

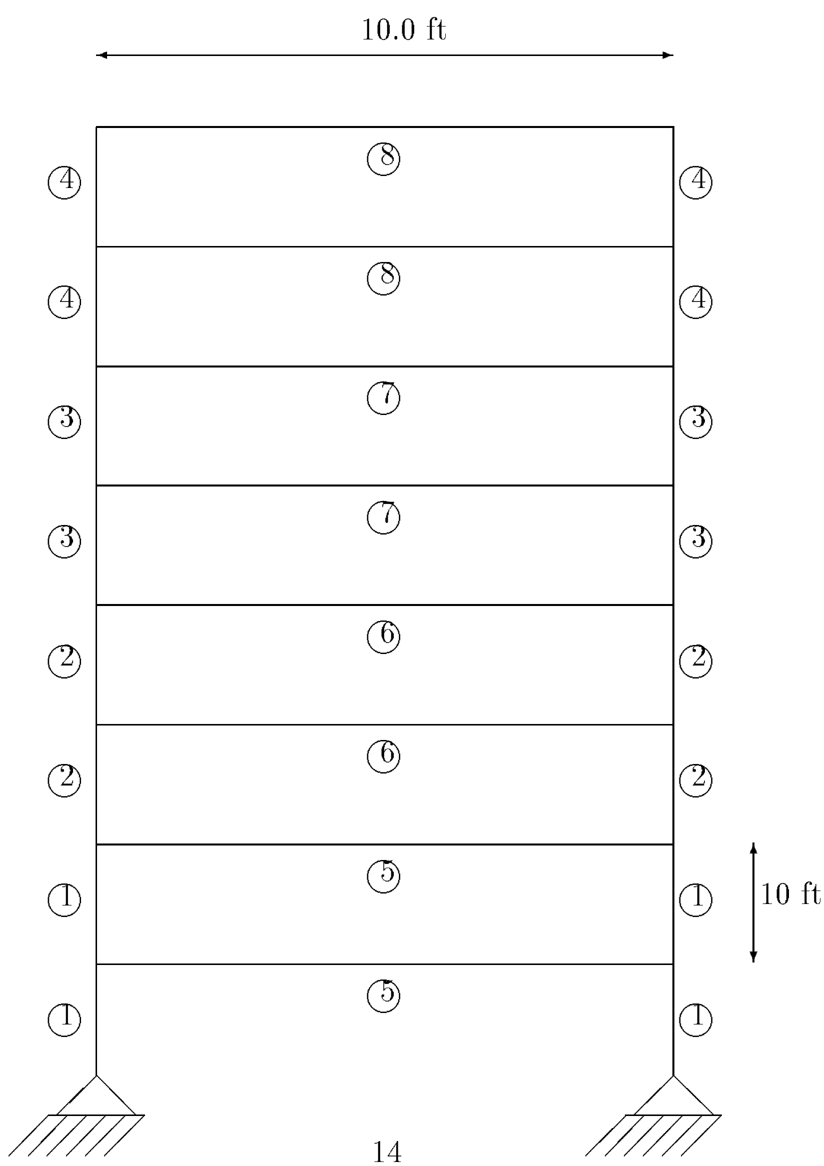

Figure 6. A one-bay eight-story frame

The constant value of F_a (see section on stress constraints) is now calculated as follows:

$$C_c = \pi\sqrt{2E/F_y} = 126.1$$

if $K\ l/r_x\ >\ C_c$ then,

$$F_a = F'_{ex},$$

else

$$F_a = \frac{[1 - \frac{(K\ l/r_x)^2}{2C_c^2}]F_y}{1.667 + 0.375\frac{K\ l}{C_c r_x} - 0.125\frac{(K\ l/r_x)^3}{C_c^3}}.$$

We also choose a more conservative value for $K = 2.6$, although a better strategy would be to extract this variable quantity from the results of the analysis code. Additionally, we scale the parameters by the initial values, so that the starting variables for PRAXIS are all equal to 1.0.

Starting from the same design as in the previous section we obtained the following admissible design with this new approach.

```
                 One bay, 8 story frame
                    Feasible designs

Design            1 Weight    11229.55

-------------------------------------------------------------------
W-sect.        Area          Moment           sx           rx
w14x68       19.97100       723.0000       103.0000       6.010000
w12x65       19.05900       533.0000       87.90000       5.280000
w12x65       19.05900       533.0000       87.90000       5.280000
w12x40       11.73700       310.0000       51.90000       5.130000
w16x26       7.670000       301.0000       38.40000       6.260000
w16x26       7.670000       301.0000       38.40000       6.260000
w16x26       7.670000       301.0000       38.40000       6.260000
w6x9         2.673000       16.40000       5.560000       2.470000
-------------------------------------------------------------------
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11 Conclusions

We have described two procedures that combine finite element structural design codes with optimization ones, in order to minimize the weight of a given structure subject to gravity and wind loads, maximum drift and maximum stress constraints.

The first process finds a feasible point if the structure is under-designed, and once all the constraints are satisfied with inequality, it reduces the weight of the structure until the most stringent constraint is satisfied with equality. Since the weight of the structure is a linear function of the design parameters (cross-sectional area of the members), we know that the optimal solution will lie on a constraint boundary (or intersection of boundaries).

The second algorithm uses an exact penalty method that seems to be more robust and extensible to more realistic, large scale structures and further design constraints than the first. Both methods were illustrated by several examples.

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