Application of Optimisation Techniques to Finite Element Analysis of Piezocomposite Devices

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Abstract- This paper describes the issues surrounding the use of mathematical optimisation techniques in the Finite Element (FE) Analysis of piezoelectric devices. From its first application to piezoelectric problems in the 1980s, FE modelling has grown to encompass full devices and broadband behaviour, capable of accurately modelling device response. Typical use of such modelling in industrial settings involves the analysis of pre-determined designs, or parameter sweeps varying such things as thicknesses or materials. While this is a viable option for small numbers of parameters, parameter space rapidly increases to encompass an unfeasibly large number of simulations. A more intelligent search method is therefore needed, and a variety of optimisation techniques have been studied over the years. We present a background to optimisation techniques, the advantages and disadvantages, and an implementation of a technique based on PRAXIS [1] to FE modelling package PZFlex [2].

I. INTRODUCTION

FE modelling is now used extensively in the biomedical ultrasound industry to validate and design new prototype piezoelectric transducers. Extensive work has been done validating the approach, and a combination of powerful software, inexpensive computers, and good material characterisation efforts has led to the possibility of excellent correlation between simulation and experiment. While this provides detailed insight into a specific design, even today most moderate to large scale models take a few hours to run to completion, limiting parameter sweeps to a small number of iterations. A method of intelligently 'homing in' on the best performance characteristics would greatly aid the design engineer.

Optimisation techniques have been studied recently to achieve efficiency in the analysis of piezocomposite devices. We detail a variety of approaches to optimisation, and their advantages and disadvantages. Following this, we document the application of a specific method based on PRAXIS, an optimisation code, combined with the PZFlex FE software package to analyse piezoelectric devices.

II. METHODOLOGY

The initial step in the combination of optimisation techniques with FE methods is to ensure confidence in a single FE model of a known structure. Accurate FE modelling requires knowledge of the physical composition of the structure, driving conditions, and accurate material properties. While the first two of these are relatively simple to know, the third, material characterisation, is somewhat more problematic, particularly for piezoceramic materials. Often considerable effort is required to determine the full set of material constants [3].

With accurate material data, results are likely to be highly accurate, however experimental verification of as many parameters as possible are essential for confidence in the model. Typical parameters for comparison include electrical impedance spectra, pressure output, surface displacement profiles, and beam profiles.

Optimisation techniques have been used to determine material characterisation. Pereyra et al [4] combined PZFlex modelling with an optimiser to establish accurate correlation between modelled and experimental impedances within 150 iterations. A comparison of initial response and final optimised response against experiment for a piezoceramic disk is shown in Figures 1 (a) and (b)

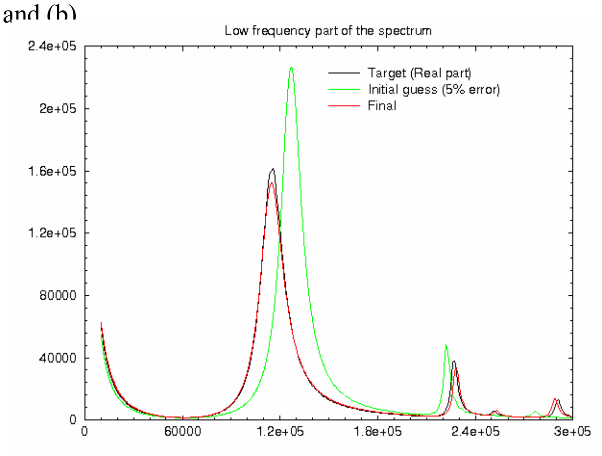

Figure 1 (a) : PZFlex Electrical Impedance Spectra compared to Experiment - Real

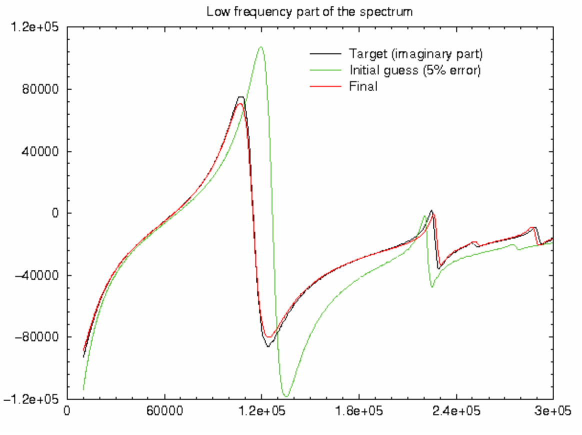

Figure 1 (b) : PZFlex Electrical Impedance Spectra compared to Experiment – Imaginary

III. OPTIMISER USE

An optimiser tries to steer a process towards a particular goal. Input parameters are varied in order to produce an output as close as possible to the stated goal function. The goal function may be any output parameter or parameters of the FE model. Examples include single values such as a pressure or a displacement, or a collection of values such as an electrical impedance spectra. The optimiser acts as a controller for the complete simulation process, deciding upon input parameters and evaluating output functions to determine appropriate changes to the input for the next iteration. The process continues until the output is sufficiently close to the goal function. This process is illustrated in Figure 2.

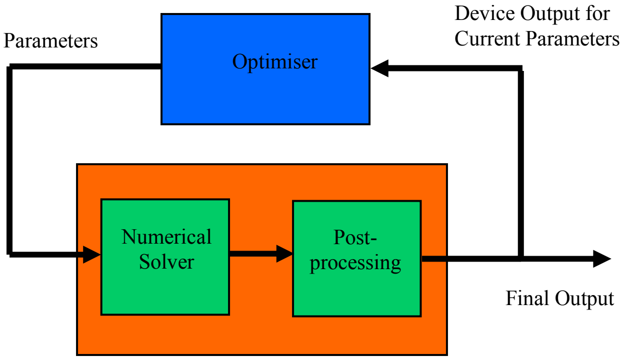

Figure 2 : Diagram of Optimiser used with Numerical Simulation Tool

The number of iterations required to achieve the best match to target function(s) varies greatly with number of variable parameters, typically ranging from a few tens of iterations to a few hundred (or more) in more complex circumstances. The data shown in Figure 1 were calculated with approximately 150 iterations, with 11 variable parameters.

Considering the number of iterations that are likely to be required, solution time of individual simulations is of great importance – a simulation that gives results within seconds will be far more amenable to hundreds of iterations than a simulation which takes hours. A solution requiring a smaller number of iterations, say 50, will take around two days to complete if the single simulation time is an hour, but less than an hour to complete should single solution time be a minute.

An alternative to this serial running of the iterations is to run multiple simulations in parallel. While this requires a greater investment in hardware to run the simulations, it is possible to have at least a linear speedup with number of parallel simulations, and if done properly a super-linear speedup. By tracking the various results in an overall database, an intelligent algorithm not only ensures the same parameter space is not investigated multiple times, but that the best search pattern is made.

We will consider the serial case as the best method of explaining the optimisation approach. A two-dimensional parameter space, such as is shown in Figure 3, details possible response minima (best target solutions). Clearly, as number of parameter variables increases, the complexity of the searchable parameter space increases rapidly.

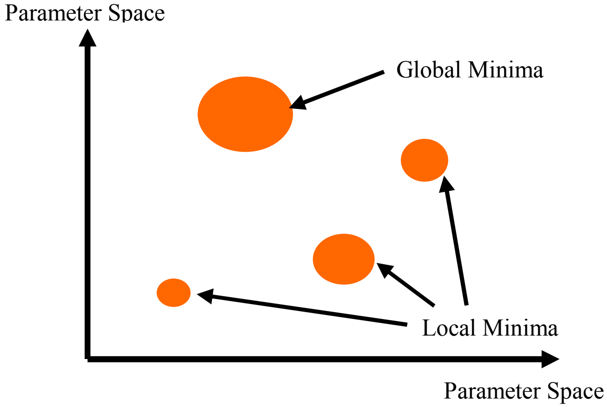

Figure 3 : Various Minima in Parameter Space

Clearly, without limitations, parameter space is infinite, and so a bounding range of parameter values are typically specified by the user to limit the searched space. Initial parameters are provided as a starting point in this space, and then early iterations make small variations of one parameter at a time, as the optimiser tries to 'learn' the response of the structure.

As data is gathered with more iterations, parameters are varied by larger quantities, and then multiple parameters are varied simultaneously. Once the optimiser has found the best fit to the target functions and parameter variations within a given range do not yield better results, the simulations are halted and the final values reported. While this usually results in the best global response, it is possible that the optimiser has selected a local minima, and there is a better solution available somewhere in the parameter space. Choice of multiple starting points for multiple simulations may yield a better response at the expense of greater computation time. The trained eye of the engineer is typically of great benefit at this point, using the optimiser to augment, and not replace, the design process.

IV. Example Application

In order to illustrate the use of an optimiser in a practical situation, and the necessary considerations, we will present the results of simulations modelling a 250kHz tonpilz transducer. The main goal is to improve on the original design by varying the various layer thicknesses, whilst keeping the same basic structure format. The desired goal was to maximise acoustic power output at 250kHz into the load medium, water.

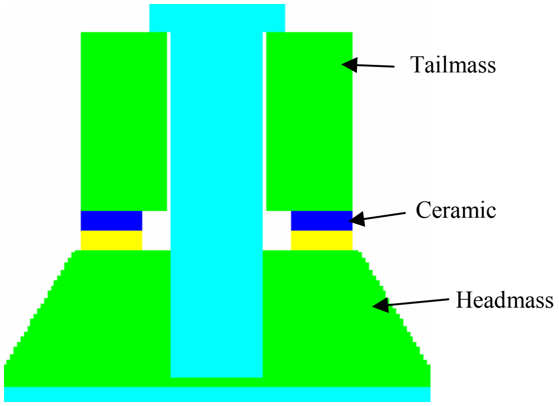

Figure 4 : Basic Tonpilz Structure for Variation

Three parameters were chosen for variation – headmass thickness, tailmass thickness, and piezoceramic thickness. While many more parameters could have been chosen, we limited the number to three to simplify the example. In addition, this particular example has been chosen to show the effect of multiple 'local minima' in the parameter space.

The initial step was to construct a reliable model in PZFlex of the basic tonpilz device. This included both the simulation itself and the post-processing required to calculate the power output. Total simulation and post-processing time was under 3 minutes on a standard desktop PC, lending itself well to potentially large numbers of iterations in a short time period. The electrical impedance and acoustic power spectra for the basic structure are detailed in Figure 5.

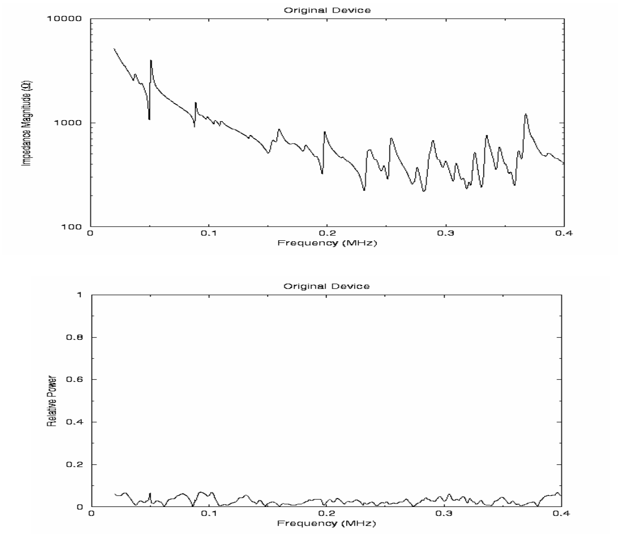

Figure 5 : Electrical Impedance and Acoustic Power Output (Normalised) for Original Tonpilz Device

For the optimisation process a goal function was required. In the interests of examining the consequences of goal function choice, five different target functions were chosen.

- Optimisation 1 - maximum power at 250 kHz relative to all other frequencies.
- Optimisation 2 - maximise total power out in the DC to 1MHz range.
- Optimisation 3 - maximum power between 160 kHz and 275 kHz.
- Optimisation 4 - maximum power between 225 kHz and 275 Hz.
- Optimisation 5 - maximum power between 225 kHz and 275 kHz relative to total power out in the DC to 1MHz range.

Each optimisation set was completed in under 40 iterations, resulting in each set taking around 2 hours to calculate, and around 9 hours computation time total for all 5 cases.

The impedance and power spectra are shown below for targets 1, 3 and 5, which resulted in the best predicted performance.

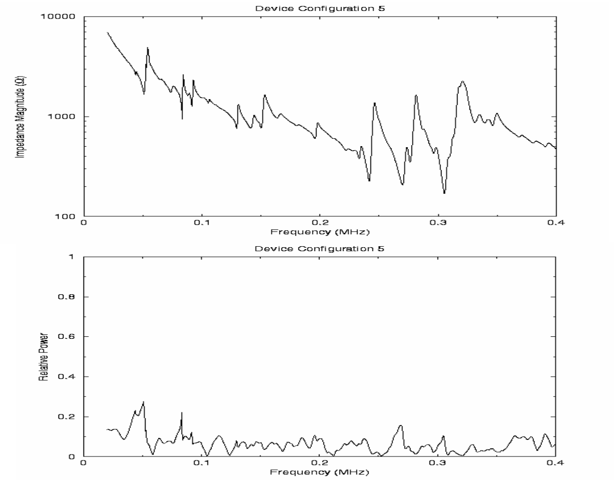

Figure 6a : Electrical Impedance and Acoustic Power Output (Normalised) for Target One

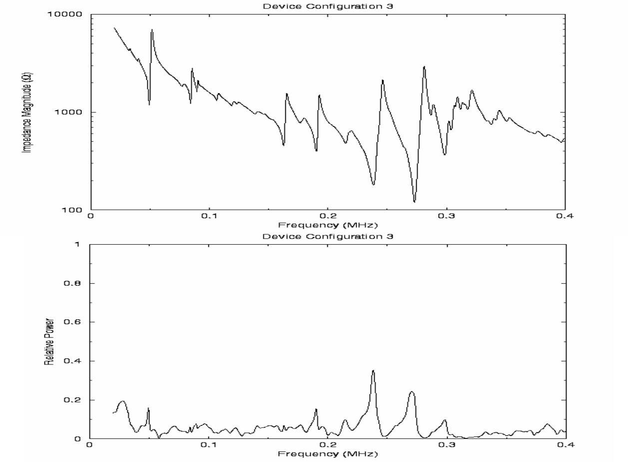

Figure 6b : Electrical Impedance and Acoustic Power Output (Normalised) for Target Three

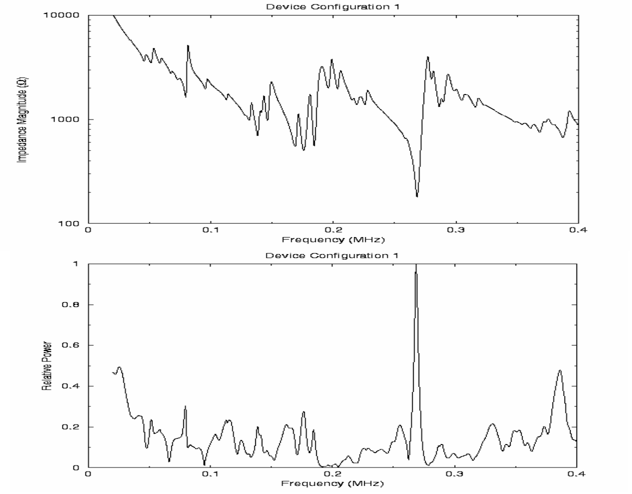

Figure 6c : Electrical Impedance and Acoustic Power Output (Normalised) for Target Five

	Base	**1**	**2**	**3**	**4**	**5**
Tailmass	23.6	24.2	13.2	23.6	9.4	5.9
Ceramic	2.5	3.4	5.2	3.6	3.9	5.1
Headmass	15.5	2.2	4.8	7.9	8.2	19.2

Table 1 : Sizes of tonpilz sections in mm, for original device and results for 5 optimisation goals.

As can be seen, choice of goal function has produced a variety in the outcomes. In effect, the choice of goal has changed the minima within the parameter space, in each case the optimiser attempting to choose the best response (global minima). Goal functions clearly must be chosen as close to the desired output as possible, with an understanding of the actual tonpilz device and its behaviour needed. For example, in the simplest case (Target One) of maximum power at 250kHz relative to all other frequencies, the optimiser struggled to maximise that value while minimising all other frequencies' output, whereas requesting maximum output power across a frequency band (Target Five) was much more realistic and attainable.

V. Conclusions

Mathematical optimisers, in conjunction with fast and accurate numerical modelling techniques, can result in an excellent design tool to aid the ultrasound engineer (or any other system that can be simulated). However multiple optimiser choices exist and care must be taken to choose one appropriate to the field of study. Even within an individual optimiser, choice of goal function is critical to obtaining the best solution, and understanding of both the optimiser and the problem is therefore required. Use of parallel processing and appropriate analysis techniques can provide significant benefits.

VI. Acknowledgements

The authors would like to thank the NSF which funded part of this research under Phase II SBIR Grant #0321420.

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