A Boundary Value Approach to Process Salt Proximity Surveys

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Summary

The vertical parts of salt domes are notoriously hard to image with seismic methods. A salt proximity survey uses receivers on wells near the salt face and shots placed in the surface on top of the salt in order to map the salt face. Travel time picks of direct transmitted events are used as data. We assume that the sediments, surrounding layers, and the top of the salt geometry are already known and available in the form of a 3D-VSP model [4]. The source-receiver rays corresponding to the picked events travel from the free surface to the top of the salt, through the salt body and emerge from it at unknown points in the salt face, reaching eventually the receivers on the wells, where they are recorded. By using three component instruments, both the total travel time and the direction of arrival at the geophones will be known. Our algorithm uses this information to determine the points of exit from the salt body, together with the normal vectors to the salt face at those points. This is then used to produce a surface patch representing the salt face. A boundary value approach is used to solve this inverse problem in complex three dimensional inhomogeneous media. No aplanatic surfaces are required. An example of application is included.

Introduction

The vertical parts of salt domes are notoriously hard to image with seismic methods. A salt proximity survey attempts to map the vertical face of a salt dome by using, in an appropriate fashion, travel time picks of direct transmitted events from a specially designed vertical seismic profiling survey. We describe in this paper a new, fast method to process a salt proximity survey in three-dimensional complex geological media. No aplanatic surfaces are necessary for this algorithm. In what follows, it is assumed that the sediments, surrounding layers, and the top of the salt geometry is already known and available in the form of a 3D-VSP model [4] Shots are set in the free surface above the top of the salt, while receivers are placed on a well (or wells) located near the vertical salt face. The source-receiver rays of interest hit the top of the salt, travel through the salt body and emerge from it at points on the unknown vertical face, reaching eventually the receivers on the wells. By using three component geophones, both the total travel time and the direction of arrival at the receivers are known. The purpose of the salt proximity survey is to use that information in order to determine the points of exit from the salt body, together with the normal vectors to the salt face at those points. It turns out that the problem of determining such information in a general piecewise inhomogeneous media can be written as a two-point boundary value problem for the ray (ordinary differential) equations (see Section 3) [2]. The numerical solution of such a problem can be accomplished by a modification of the general ray solver **RAY3D** [3]. **RAY3D** uses an iterative procedure to solve a coupled systems of nonlinear difference equations, and therefore it requires an initial discrete trajectory. We generate such a trajectory by means of a shooting procedure that will be described in the next section. Once we know how to solve for the ray corresponding to a source/receiver pair we will need only to loop through all the source/receiver pairs associated with the collected data in order to obtain a number of points on the salt vertical face. Within the same process we will calculate the normal to the unknown salt surface at the points of exit of the rays. Finally, by using the procedures described in [1], we can process this surface data in order to incorporate it into the existing 3D-VSP model, for further processing.

Shooting procedure to initialize the two-point solver

Figure 1 shows the schematics of a typical salt dome structure, with sediments, lateral layers of the various rocks pierced by the salt, an unmapped vertical face, and a vertical well. It also shows the sketch of a source-receiver configuration and the corresponding ray paths. Observe that this is not a valid *georay* model; because of the missing salt wall, the salt body is not completely bounded. However, this will not affect our algorithm, provided that the layers pierced by the dome are terminated properly. If these layers are truncated too inaccurately, then there may be some inconsistency for rays that arrive close to the layer/salt intersection. The shooting procedure has two separate parts. First we shoot a fan of rays starting at the source. The initial directions of these rays should be such that they arrive to the top of the salt with a sub-critical direction, so that the ray can be transmitted through the salt. This transmitted ray should point in the general direction of the salt wall where the ray from the receiver will arrive. The second part of the procedure consists of shooting a ray from the receiver in a direction that is the negative of the measured arrival direction. This ray is completely determined by these conditions, until it reaches the unknown salt face. At each step of the shooting procedure we verify if there has been a change of sign in the function:

$$\begin{aligned} f(s) &= t_a - (t(s) + t_\nu \\ &\quad + t_{salt}(s)) \end{aligned} \tag{1}$$

where

t_a Total measured travel time.
$t(s)$ Current elapsed time for shot from the receiver.
t_ν Time from source to top of salt.
$t_{salt}(s)$ Time within the salt along the straight line $\overline{\eta_{\mathbf{top}}, \eta_{\mathbf{face}}(\mathrm{s})}$,

where

$\eta_{\mathbf{top}}$ is the point of intersection of the source ray with the top of the salt, and
$\eta_{\mathbf{face}}(\mathrm{s})$ is the current position of the ray shot from the receiver.

The change of sign in the travel time function $f(s)$ will indicate that we have (approximately) traversed the salt wall, at some point between the current point $\eta_{\mathbf{face}}(\mathrm{s})$ and the one from the previous step. Once this change of sign is located, we call ZEROIN, a zero finding routine, to obtain a more precise intersection. The resulting point then replaces $\eta_{\mathbf{face}}$. By taking small steps we expect that the point $\eta_{\mathbf{face}}$ will be a reasonable approximation to the ray-salt face intersection; with that point calculated we can assemble an approximate trajectory to initialize the two-point procedure (that will be described in the next section), by joining the ray from the source (R_1) with the straight line segment $\overline{\eta_{\mathbf{top}}, \eta_{\mathbf{face}}}$ that represents the trajectory through the salt body (S), and setting up an appropriate detailed signature. Observe that this is not a ray, because of the arbitrary leg within the salt. If the two-point procedure succeeds, then we are done, otherwise we return to shoot rays from the source, until we find one that arrives at the top of the salt in a subcritical direction.

Two-point procedure

Once an initial ray has been secured, we can switch to a two-point or bending type algorithm, in order to obtain a proper ray between the source and receiver, determining also the unknown ray-vertical salt face intersection point and the normal to the salt face there. Since the ray shot from the receiver is fully determined until it reaches the unknown salt face, we will establish our boundary value problem only for the portion of the ray that goes from the source to the unknown salt face ($R_1 + S$). The assumption, valid for constant media and away from sedimentary interfaces, is that the unknown ray-salt face intersection will lie along the line that joins the two last points in R_2. In other words, the end point of the two-point ray η_n satisfies:

$$\eta_{\mathrm{n}} = {\eta_{\mathbf{face-1}}}^{R_2} + c[{\eta_{\mathbf{face}}}^{R_2} - {\eta_{\mathbf{face-1}}}^{R_2}] \qquad (2)$$

where the points $\eta_{\mathbf{face-1}}$ *and* η**face** in the trajectory R_2 are the points defined above. Obviously, the scalar c is unknown, and it will have to be determined during the solution of the two-point problem. We can initialize it to $c_0 = 1.0$, say. To analyze and define the modified boundary value problem, it is enough to consider a simplified situation in which there are no sediments and the media is homogeneous outside the salt (see Figure 1). Thus, the desired ray, traveling from the source, refracting on the top of the salt and arriving to the unknown salt face, will have only two legs and a total of four mesh points. Since the ray equations [3] have seven unknowns per mesh point, namely:

η_i the ray position vector;
ω_i the ray direction vector, scaled by the slowness;
t_i the partial travel time,

we will have twenty eight unknowns from this source. Then we have to count also the unknown arc lengths along each leg, which give two additional quantities to be determined. On the other hand we have fourteen difference equations, one arc lenght condition, four initial conditions from the source position and from setting the initial time to zero; seven interface conditions at the top of the salt (continuity and Snell's law), and a condition to insure contact with the top of the salt interface, for a total of twenty seven equations. The interesting new condition occurs at the unknown salt interface. We have the direction of arrival from the receiver (see (2)), while the unknown point on the salt face is parametrized by c. Adding this parameter to the list of unknowns listed above gives a total of thirty one variables to be determined. The missing four additional conditions are then given by (2) and by the travel time equation (1). Observe that in this way we need to calculate only a partial two-point ray, since the trajectory from the receiver to the salt face is essentially fixed and can be calculated once and for all by the shooting procedure. Adding sediments, including inhomogeneous regions, both to the top and to the salt flanks, does not introduce any additional complexity, since those structures are already contemplated in the standard RAY3D algorithm. After a succesful two-point ray has been calculated we can then assemble a full source-receiver trajectory by adding to it the appropriate piece of R_2, using for this purpose the final value of the parameter c. The vector η_n will be on the salt face, and to determine the normal we can use again Snell's law, now with prescribed incoming and outgoing directions:

$$\omega_T = \omega_I + \gamma * \mu_{salt}, \qquad (3)$$

where T = *transmitted*, I = *incident*, μ_{salt} is the unknown normal vector, and the scalar $\gamma = \sqrt{u_T^2 - u_I^2 + < \omega_I, \mu_{salt} >^2}$. We observe from this equation, that the direction of the normal is equal to the difference of the incident and transmitted (scaled) directions. In other words, the normalized normal to the salt face at the point η_n is given by:

$$\mu_{salt} = (\omega_T - \omega_I) / \parallel \omega_T - \omega_I \parallel . \qquad (4)$$

Conclusions

We have presented a boundary value solution to the problem of finding the salt face exit points in a salt proximity survey. The ray tracing based method is very fast, reliable, works in complex three-dimensional media, and it requires no aplanatic surfaces. In Figure 2 we show an example of a multiwell survey processed using this technique.

References

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