Improving a bad static problem in the Arapahoe field by 3D nonlinear travel time inversion

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Abstract

The first 1200 feet below the surface of the Arapahoe field in the Colorado-Kansas border contain a number of embedded low velocity sand bodies that constitute a severe problem for static corrections. We describe the application of a modeling and inversion system to map these low velocity anomalies so that better imaging can be obtained below them. Weidlinger Associates **INTEGRA,** is a forward and inverse three-dimensional modeling system that can deal with complex geological structures. It has been designed to handle large scale problems by using a distributed approach, it uses seismic ray tracing for forward simulation, and nonlinear travel time inversion for imaging. A novel decomposition method is the tool to attack large scale problems in a parallel approach. It is fully implemented and operational, as we demonstrate here on a real data set.

Introduction

The problems that our system can address are those related to the inversion of picked, interpreted, pre-stack travel times of selected events, to obtain the parameters describing a blocky three-dimensional model of an earth volume. A blocky model is defined as an aggregate of irregularly shaped rock volumes bounded by surface patches. The rocks within a region can be inhomogeneous, but they are assumed to be slowly and smoothly varying, while sharp material discontinuities are explicitly modeled by the interfaces.

The objective of the travel time inversion process is to improve upon the accuracy of both reflector positions and inhomogeneous velocities within the regions. Since these problems are usually very large and computationally intensive, we use an approach that leads naturally to coarse grain parallelization in a distributed computing environment.

We attack this task with a complete set of forward modeling tools that includes interactive structural model construction and 3D seismic ray tracing. The forward modeling system can simulate all the important data acquisition modalities, namely: surface reflection and refraction, cross-well, VSP and inverted VSP. In other words, sources and receivers can be in the surface of the earth or down holes. Reflection, as well as transmission data can be handled, even for cross-well acquisitions. Mode conversions, a transverse isotropic option, and diffractions by curved edges are also included.

The most practical parametrization that we use, both for surfaces and volumetric material properties, involves tensor products of cubic splines. A B-spline representation has the advantage of being local, i.e., changes on a weight associated with a basis function has only a local effect on the value of the surface or volumetric property represented. This representation is also automatically smooth and meshes or triangulations of any resolution can be easily generated as needed. The ray tracing takes into account full curved rays in 3D. Very careful and complete ray theory amplitudes are also available.

Still, for 3D complex models, the number of parameters necessary for an accurate representation may be large, precluding the use of direct Singular Value Decomposition (SVD) based solvers for the linearized, ill-conditioned least squares problems that arise when nonlinear travel time inversion is performed with a Marquardt type algorithm. The alternative route, that most researchers in this area have taken, is to consider iterative methods of the conjugate gradient type, while making some approximations to restrict the amount of ray tracing required.

In our approach, we like to preserve the desirable feature of using SVD based analysis. To achieve this objective we will show how to break the problem into blocks in a natural, problem oriented fashion. As a bonus we will obtain an algorithm that is easily paralellizable on an ensemble of processors.

The algorithm has two phases. In the pre-processing phase we analyze the linearized problem at an initial model guess. This linearized model is represented by the Jacobian matrix of first derivatives of travel time with respect to model parameters. The derivatives are calculated exactly by solving the linearized equations with appropriate right hand sides, so they include the variation of velocity with respect to changes in the ray path.

We subdivide the set of observations in subsets, clustering together those observations whose ray paths share the same localized volume of model space. For each one of these data subsets we use a quantitative criterium to determine the model parameters that are most relevant to it.

After completing this pre-processing, we would have produced a partitioning of both observations and model parameters into smaller sub-problems. We stress the fact that different parameter sub-sets may overlap, and that in general we would have neglected some weaker couplings between different subsets; i.e., not all parameters relevant to a data set are necessarily included, but rather some threshold is used (clearly, if there were no couplings and/or overlaps between blocks the sub-problems could be solved independently).

The second phase of the procedure consists of a block Gauss-Seidel iteration, in which nonlinear least squares problems are solved for the parameters in each block, maintaining the values of the other parameters fixed. In a sequential approach, these solves would proceed in a given order, updating the vector of model parameters after each block solve. In a parallel, asynchronous approach, multiple processors tackle different blocks at the same time, updating a central copy of the model parameters as they finish their alloted task. A Jacobi approach is also possible, in which the update of the master parameter vector is only done after all the block solves have been completed. This is also a synchronization step, and it allows to calculate the updated values of the parameters as a weighted average of all the block values (in case of overlaps).

By keeping the number of parameters in each block small (say less than 100), we can still solve the individual nonlinear least squares problems with a Marquardt/SVD based algorithm that facilitates regularization. We observe that, as usual, the objectives of model fitting and regularization are conflicting: if we include weakly determined parameters in a sub-model, then the problem will be more ill-conditioned. Thus, the parameter acceptance threshold is in itself a regularization knob for the blocks.

We demonstrate this complex set of algorithms and computer programs on the Arapahoe field data set, for which the goal is to improve an existing velocity model in order to account for large low velocity anomalies which distort the pre-stack migration of the reflection seismic data. The data we use is a fairly large set of reflection travel time picks in 3D.

The detailed description, theoretical aspects, and application of our methodology to synthetic problems have been published elsewhere [5, 6, 7, 8].

Block inversion pre-process

The pre-processing for converting a large problem into block form starts by reading the time data and subdividing it. Currently, we assume that the data consists of a number of shots and reflections from various interfaces. Transmission data as in VSP, inverted VSP or cross-well configurations can also be handled. If we assume that the receiver array corresponding to each shot is fairly localized, i.e., has small aperture, so that the beam associated with the shot "sees" only a small part of the earth volume under study, which in turn is described by a small number of parameters in the model, then it is natural to subdivide the data set into its individual shots. Clearly other subdivisions are also possible (see for instance (Kennett et al, 1988)).

As a matter of fact, in some applications we have implemented a hierarchical subdivision. First we subdivide in sub-tasks corresponding (say) to different acquisition geometries, and then each sub-task is further subdivided into its component shots. The pre-processor also produces automatically all the necessary data files for ray tracing and inversion in each block sub-problem.

Having done that subdivision, we proceed to analyze the data to determine which parameters are actually well determined by each subset of data.

During the data analysis phase program BLKINV makes a system call to program FULINV, in such a way that only the first iteration of the nonlinear optimization process is effected. Then, after having traced and matched rays with observations for the initial guess, it proceeds to calculate the SVD of the Jacobian matrix of the residual vector. This potentially very large matrix is trimmed by eliminating all zero columns. Finally, FULINV outputs the V^T part of the Singular Value Decomposition for BLKINV to complete the analysis. The details of the analysis and the validation of the method on synthetic examples have been discussed elsewhere [7].

Real data application

We consider now the application of these methods to a large scale three-dimensional surface reflection survey. The purpose is to provide a better velocity for a highly inhomogeneous weather layer that is creating problems for imaging below it. This is a static correction problem, and a more accurate velocity that includes suspected low velocity lumps would help.

An indication of the anomalies is given by a two-way normal incidence travel time map across the region under exploration (Figure 1). This map shows strong variations in travel times: from 350 to 550 miliseconds within this approximately 1,200 ft. thick layer. The low velocity areas show as well defined bodies, mostly in the northern part of the region. We are also given several velocity profiles constructed from well logs that show a piecewise linear in depth background velocity.

Because of the limitations of the inversion of small aperture reflection data to good resolution only in the lateral directions, we constrain our velocity model to be a lateral correction to a fixed vertical velocity obtained as a weighted average of the well velocities.

The region under study has an extension of $49,600 \times 46,500 \times 1,200\ ft^3$. We work with approximately 250,000 travel time picks corresponding to more than 7,100 shots, and including primary reflections from the Fort Hayes interface. We have also used digitized maps of the reflectors that were readily incorporated into a structural model via B-spline least squares fitting. These reflectors are already well determined and they are not the object of this exercise.

We have been able to solve this problem in a reasonable amount of time because the system is fully parallelized under PVM (Parallel Virtual Machine). We have been working on applications of distributed computing to geophysics since 1991 [4], and that experience has been invaluable in this project. We have run this problem on a 100Mbit non-dedicated Ethernet linking an heterogeneous network involving three different operating systems and six CPU's: two Pentium Pros, 200 MHZ, one Pentium II, 300 MHZ, all running Solaris for x86, one Sparc II running SUN OS, one Sparc 10, and one Sparc 5 running Sun Solaris. In order to improve the parallel performance we assigned more that one task to each of the fast processors, thus counteracting network and disk I/O latency.

It took approximately 101 hours of CPU time to complete a full sweep over the entire data set, but only less than 22 hours of wall clock time, for a parallel speed up factor of about 4.6. The system has re-starting capabilities and its performance scales linearly with the number of processors. It has been ported to most UNIX platforms and it has been demonstrated at the 1995, 1996 and 1997 SEG conventions on a 20 processor UltraSparc SUN system, an 8 processor Hewlett-Packard-Convex machine, and a cluster of Fujitsu HAL machines.

The travel times are given on individual records which include also the source and receiver coordinates. No explanation of the acquisition geometry was provided, so we decided to use our Generalized Lines approach. We identify first all the arrivals corresponding to a shot and then sort them as generalized lines, which are basically logically connected sequences of receivers. When ray tracing, we use these lists to do receiver continuation. Each shot will constitute one task in our multi-task approach. It will also represent a block of observations for the nonlinear travel time inversion.

The initial velocity we use is of the form:

$$v(x, y, z) = 3.05 + 4.0815z + B(x, y),$$

where $B(x, y)$ is a tensor product bi-cubic B-spline defined by a 59×59 grid of basis functions. We arrived to this mesh through a multiresolution analysis (multigrid inversion), where we started first with a coarser mesh and refined successively to reach the current level. We set all the weights in the B-spline representation to zero, so that the initial velocity is equal to the background gradient, which is kept fix through the inversion process. The initial background velocity at 600 ft. is shown in Figure 2.

The initial model, called *frhs*, consists of a composite Coons/B-spline patch representation of the topography, the reflector, and an appropriate velocity constructed from the available data. We also create a mesh of bins covering the (x, y) extension of our region, and select one shot in each bin (if available). In this way we can decimate the data set, if necessary, without sacrificing a good coverage.

We use smoothing in the form of a penalty term in the linearized least squares step of the Marquardt iteration that is employed for solving the nonlinear least squares problems corresponding to each block. The smoothing constraint consists of the discrete Laplacian of the B-spline control vertices, as advocated in [2]. This is simpler than imposing a penalty on the second derivatives of velocity, and it has the same effect, as demonstrated in the report just cited.

Finally, we also delete all observations that have offsets larger than 3,000 ft. This is done because, according to the data owners, observations associated with large offsets are less trustworthy in this data set. Additionally, an automatic outlier removal procedure deletes any observation with travel time residual that is more than a factor times the standard deviation away from the current running mean.

We provide graphical output of various kinds, both at the start of the process and after a block Gauss-Seidel sweep. A gray scale contour plot of a horizontal velocity slice at 600 ft depth provides a way to see the effect of the lateral correction, as shown in Figure 5. We also provide plots of the (more than 7,200) rms corresponding to each block (shot). These are in the form of scattered plots, showing the distribution and magnitude of the errors at the outset and after a full correction sweep. They are in the same scale, for purposes of comparison. We see that most of the residuals have been reduced well below 15 ms, which is quite an acceptable fit given the data accuracy.

As a final validation of the tomographic result, we use the corrected velocity to run a Normal Incidence ray tracing over the whole exploration area, and use the calculated travel times to create a time map which is shown in Figure 7. For comparison we also show the NI travel time map corresponding to the initial velocity in Figure 4. An additional sweep was run producing little change. In later work we expect to use additional reflections from deeper structure to see if we can increase resolution.

Conclusions

We have described an integrated forward and inverse three-dimensional modeling system that can deal with complex geological structures and acquisition geometries, and large scale problems by using a distributed approach. It uses seismic ray tracing for forward simulation, time-to-depth mapping, and nonlinear travel time inversion. A novel decomposition method is the tool to attack large scale problems by using parallel computation.

The system is fully implemented and we have demonstrated its performance earlier on synthetic examples showing its capabilities for integrating heterogeneous data sets coming from a variety of surveying approaches. In the current real data problem, we have shown that we can actually deal with a large scale data set and produce a lateral velocity correction that correlates well with the suspected anomalies, as given by two way normal incidence travel times. Although this problem is structurally simple, it presents severe lateral inhomogeinities, which the method seems to have resolved adequately. It shows also the way in which similar static problems, such as those presented by permafrost and basalt could be handled.

References

[1] Bartels, R., Beatty, J.C., and Barsky, B.A. *An Introduction to Splines for Use in Computer Graphics and Geometric Modeling* (Morgan-Kauffman Pub., Los Altos, CA 1987).

[2] Paul H.C. Eilers, and Brian D. Marx (1994), "P-splines, the ultimate smoothers?," Manuscript.

[3] Kennett, B.L.N., Sambridge, M.S. and Williamson, P.R. (1988), *Subspace Methods for Large Inverse Problems with Multiple Parameter Classes,* Geophysical Journal, 94, 237-247.

[4] Koshy, M., Pereyra, V., and Meza, J.C., *Distributed Computing Applications in Forward and Inverse Geophysical Modeling* In *Expanded Abstracts* (SEG 61st Annual Meeting, Houston, TX. 1991) 349-352.

[5] Pereyra, V. (1992), *Two Point Ray Tracing in General 3D Media*, Geophysical Prospecting, 40, 267-287.

[6] Pereyra, V. (1993) *Parallel Block Inversion of Geophysical Data.* Proc. of Mathematical methods in Geophysical Imaging. SPIE, 2033, 192-198.

[7] Pereyra, V. (1996) *Modeling, Ray Tracing, and Block Nonlinear Travel Time Inversion in 3D* , Pure and Applied Geophysics, 148, 345-386.

[8] Pereyra, V. (1998) *Asynchronous Distributed Solution of Large Scale Nonlinear Inversion Problems.* To appear in Special Issue of Applied Numerical Mathematics. V. Pereyra and J. Castillo, Editors.